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Vortex shedding from spheres

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Vortex shedding from spheres has been studied in the Reynolds number range $400 < Re < 5 \times 10^6$. At low Reynolds numbers, i.e. up to $Re = 3 \times 10^3$, the values of the Strouhal number as a function of Reynolds number measured by Möller (1938) have been confirmed using water flow. The lower critical Reynolds number, first reported by Cometta (1957), was found to be $Re = 6 \times 10^3$. Here a discontinuity in the relationship between the Strouhal and Reynolds numbers is obvious. From $Re = 6 \times 10^3$ to $Re = 3 \times 10^5$ strong periodic fluctuations in the wake flow were observed. Beyond the upper critical Reynolds number ($Re = 3{\cdot}7 \times 10^5$) periodic vortex shedding could not be detected by the present measurement techniques.

The hot-wire measurements indicate that the signals recorded simultaneously at different positions on the 75° circle (normal to the flow) show a phase shift. Thus it appears that the vortex separation point rotates around the sphere. An attempt is made to interpret this experimental evidence.

1. Introduction

Unsteady flow past spheres has been the object of numerous examinations. The flow conditions at low Reynolds numbers, in particular $Re < 1000$, Re being based on the sphere diameter d and undisturbed velocity U_∞, were of great interest with regard to problems of meteorology and chemical processes. Thus, there is already some information available on properties of the wake and on theoretical considerations as to the stability of three-dimensional vortex configurations for this flow range. Experiments carried out at Reynolds numbers greater than $Re = 1000$ are reported by only a few authors. Möller's (1938) tests in water covered Reynolds numbers up to $Re = 10^4$. Cometta (1957) investigated the unsteady flow past spheres up to $Re = 5 \times 10^5$, but he could detect periodic separation of vortices up to $Re = 4 \times 10^4$ only under restricted conditions. In recent papers Mujumdar & Douglas (1970) as well as Calvert (1972) report on vortex shedding from spheres in the Reynolds number ranges

$$5{\cdot}6 \times 10^3 < Re < 1{\cdot}16 \times 10^4 \quad \text{and} \quad 2 \times 10^4 < Re < 6 \times 10^4,$$

respectively.

Torobin & Gauvin (1959) reviewed the experimental and theoretical work dealing with the wakes of spheres. They gave a nearly complete list of authors who had worked in this field. The experimental results of several authors seem to be contradictory. While, for example, Möller (1938) measured at $Re = 10^4$ a Strouhal number of $S = 2\cdot0$, where S is given by $S = fd/U_\infty$ and f is the vortex shedding frequency, Mujumdar & Douglas (1970) reported a value lower by one order of magnitude, viz. $S = 0\cdot2$. With a view to this discrepancy, the study of Cometta (1957) is of interest since, with Reynolds numbers lower than $Re = 7400$, he detected the co-existence of low Strouhal numbers ($S = 0\cdot2$) and a higher mode ($0\cdot8 < S < 1\cdot4$). No explanation of this phenomenon is given by the author, but he suggests that a transition occurs in the vortex sheet from laminar to turbulent flow.

This was the state of knowledge when the present experiments on unsteady flow past spheres were started. The aim was to obtain information about the periodic boundary-layer separation at high Reynolds numbers and to find out how the vortices are discharged from the sphere. The Reynolds number and Strouhal number were varied not only by varying the velocity U_∞ but also by using spheres of different diameter. Thus, the low Reynolds number range was also covered, since spheres of diameters down to $d = 0\cdot02$ m were tested. At Reynolds numbers lower than $Re = 6 \times 10^3$, vortex shedding could no longer be detected by means of the measurement technique applied. Therefore, some tests were carried out in a water tunnel to render the flow visible. In this test facility, which was available from previous investigations, a maximum Reynolds number of only $Re = 3 \times 10^3$ could be reached. Therefore, no experimental results are available in the intermediate range $3 \times 10^3 < Re < 6 \times 10^3$. In the upper Reynolds number regime, tests were run up to $Re = 5 \times 10^6$. Immediately before the critical Reynolds number $Re_c = 3\cdot7 \times 10^5$ was reached, the periodic signals were no longer indicated by the hot wire and did not occur again before $Re = 5 \times 10^6$.

2. Test arrangement and measurement techniques

The flow-visualization experiments were carried out in a water channel with a diameter $D = 0\cdot25$ m. The maximum velocity of the incident flow was $U_\infty = 0\cdot09$ m/s. Spheres with $d = 0\cdot02$ m and $0\cdot04$ m were tested. They were supported from the rear by a sting, which had a length $0\cdot2$ m and a diameter $0\cdot1d$. At an angle $\phi = 40°$, measured from the front stagnation point, dye was introduced into the boundary layer through a hole in the sphere with a diameter $d_h = 0\cdot5 \times 10^{-3}$ m. Thus, the rolling-up of the shear layer separating from the sphere could be observed with the naked eye.

The experiments in the Reynolds number range $6 \times 10^3 < Re < 3 \times 10^5$ were performed in a wind tunnel using air at atmospheric conditions. The turbulence level was about $0\cdot45$ %. The spheres supported from the rear were put into the free air jet downstream of the nozzle, which had a diameter $d_n = 0\cdot75$ m. Experiments at Reynolds numbers greater than 3×10^5 were carried out in a high-pressure-wind tunnel, which had the same internal dimensions as the atmospheric wind tunnel. A Reynolds number $Re = 5 \times 10^6$ could be reached using a system pressure of 40 bars.

The frequency of the unsteady boundary-layer separation was detected

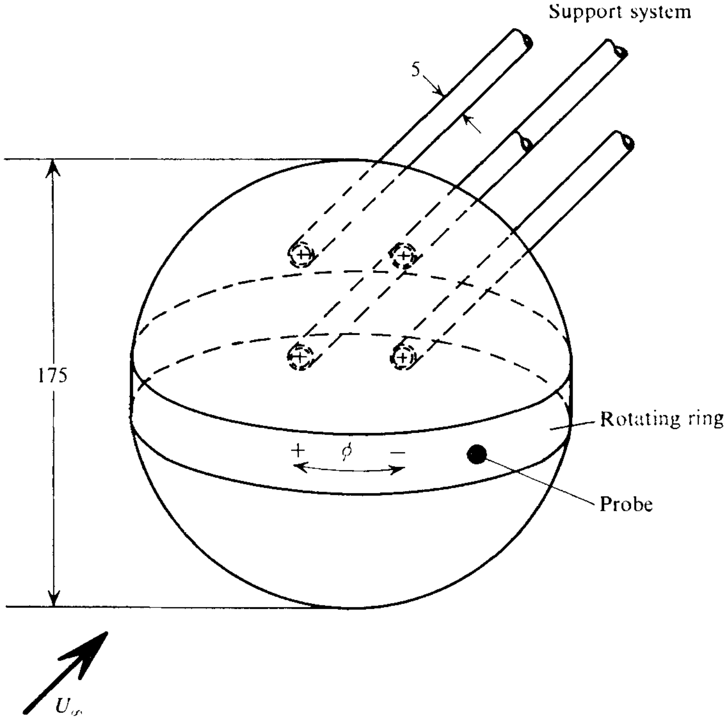


FIGURE 1. Test sphere with movable hot-wire probe, dimensions in mm.

by means of hot wires. It proved advantageous to install the probes flush with the surface of the sphere. Thus, displacement of the hot wire relative to the sphere, caused by vibrations, was eliminated. From previous investigations on the local skin friction and static pressure distribution around spheres (Achenbach 1972) a test sphere with $d = 0{\cdot}175$ m was available (figure 1). The hot wire was mounted on a ring, which could be rotated from inside by an electric motor. In this way, the position of the optimal signal could be easily detected. It was found that the most intensive signal was obtained about 7° upstream of the boundary-layer separation point. In the subcritical Reynolds number range, where the separation occurs at about $\phi_s = 82°$, this optimal position is located at about $\phi = 75°$. On the basis of this evidence, five test spheres, with $d = 0{\cdot}198$, 0·175, 0·133, 0·076, 0·040 and 0·020 m, were fitted with hot wires at $\phi = 75°$.

At low velocities and large sphere diameters, the vortex shedding frequency dropped down to 2 Hz. Because the analyser had a minimum frequency of 20 Hz, a frequency transformation by means of a tape recorder was required. The transformation ratios used were 1:4, 1:10 and 1:40.

3. Results

3.1. *Tests in a water channel*

The rolling-up of the shear layer separating from the sphere was observed in a water channel. At $Re = 400$, the resultant vortex sheet began to form loops, which were periodically released from the sphere. The vortex shedding

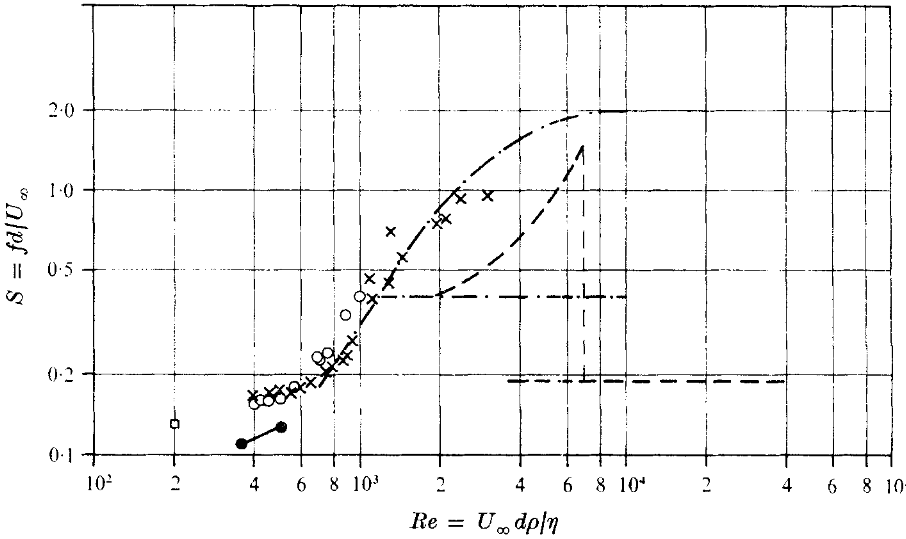


FIGURE 2. Strouhal number *vs*. Reynolds number for spheres. ·····, Möller (1938); - - - -, Cometta (1957); ●—●, Magarvey & Bishop (1961); □, Marshall & Stanton, normal plate. Present results: ○, $d = 0{\cdot}02$ m; ×, $d = 0{\cdot}04$ m.

frequency was determined by measuring the time necessary for a sequence of 50 vortices to be released. Each experimental result, shown in figure 2, represents the mean value of six runs. The scatter around the mean value was generally less than $\pm 5\%$. Figure 2 shows that the present results fit the mean curve representing the results obtained by Möller (1938) very well. The results reported by Magarvey & Bishop (1961) are not confirmed satisfactorily. This discrepancy may be caused by the different experimental methods: Magarvey & Bishop used falling spheres in liquid at rest, whereas, in the present case, a rigidly supported sphere was tested in a moving fluid. A freely falling sphere can react to changing flow forces by displacement, whereas a rigidly supported sphere cannot; thus the hydrodynamic forces as well as the formation of the wake can be different for the two cases.

For comparison the experimental results of Cometta (1957) are also plotted. His values for the higher frequency mode of Strouhal numbers are lower, by approximately a factor of 2, than those of Möller and the present ones.

3.2. *Tests in wind tunnels*

The experiments carried out in the atmospheric and high-pressure wind tunnels extended from $Re = 6 \times 10^3$ to 5×10^6. The dependency of the Strouhal number on Reynolds number in the subcritical flow regime, i.e. $Re < 3 \times 10^5$, is shown in figure 3. The length scale, i.e. the diameter d of the spheres, was varied by a factor of 10. In the range $Re = 6 \times 10^3$ to 3×10^4 the Strouhal number rises from $S = 0{\cdot}125$ to $S = 0{\cdot}18$. Up to $Re = 2 \times 10^5$ only a small increase (to $S = 0{\cdot}19$) is observed. Another small increase (to $S = 0{\cdot}20$) occurs just before the critical Reynolds number is reached. In the Reynolds number range covered by the

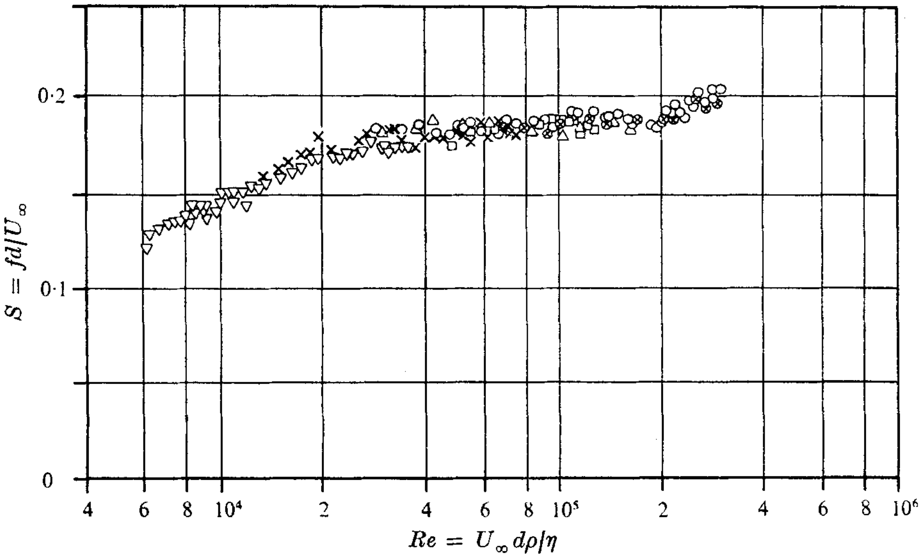
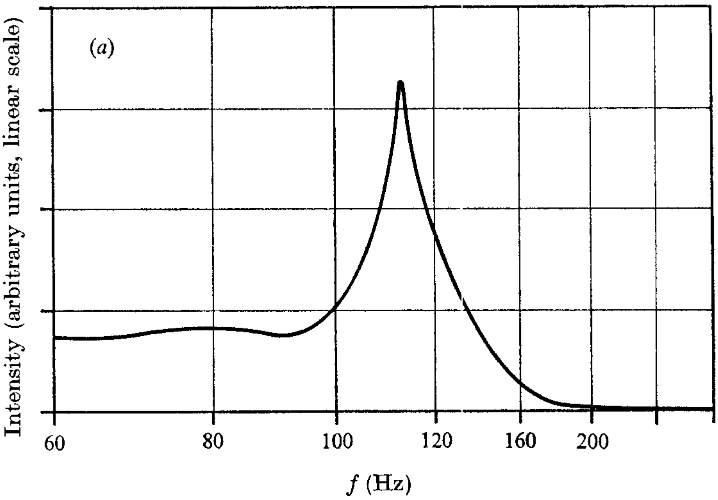


FIGURE 3. Strouhal number *vs*. Reynolds number for spheres. ▽, d = 0·020 m; ×, d = 0·040 m; □, d = 0·076 m; △, d = 0·133 m; ○, d = 0·175 m; ⊗, d = 0·200 m.

results given in figure 3, the periodic boundary-layer separation was indicated by a sinusoidal signal from the hot wire. The signal represents a measure of the temporal fluctuations of the local skin friction immediately upstream of the separation point. The hot-wire signals were analysed in a narrow band frequency analyser. At the vortex shedding frequency the spectrum showed a significant intensity peak as illustrated, for example, in figure 4 (*a*). For Reynolds numbers smaller than $Re = 6 \times 10^3$ neither this peak nor any other could be detected by the measurement techniques used since the signal disappeared abruptly. It is possible that the mechanism of boundary-layer separation changed. Unfortunately, the investigations could not be extended into the wake flow to confirm, perhaps, the high values of the Strouhal number reported by Möller (1938) and Cometta (1957).

Above $Re = 3 \times 10^5$, the sinusoidal hot-wire signal again vanished abruptly. At this value, the critical Reynolds number is approximately reached as is known from previous investigations (Achenbach 1972). Here the pressure drag coefficient drops rapidly because of a downstream shift of the boundary-layer separation point. The boundary layer undergoes transition from laminar to turbulent flow at a position which is a function of the Reynolds number. Though the probe was rotated around the entire circumference of the sphere no prevailing frequency could be detected into the whole range $3 \times 10^5 < Re < 5 \times 10^6$. Figure 4 (*b*) shows that the spectrum picked up at the surface of the sphere is broad compared with that for subcritical conditions (figure 4*a*). This experimental evidence was also confirmed by measurements of the autocorrelation function $\rho_x(\tau)$, defined below [equations (2) and (3)]. The periodicity of the wake flow is easily seen for the subcritical case (figure 5*a*). A Strouhal number of $S = 0{\cdot}183$ was calculated from the wavelength of the curve, which in this case represents



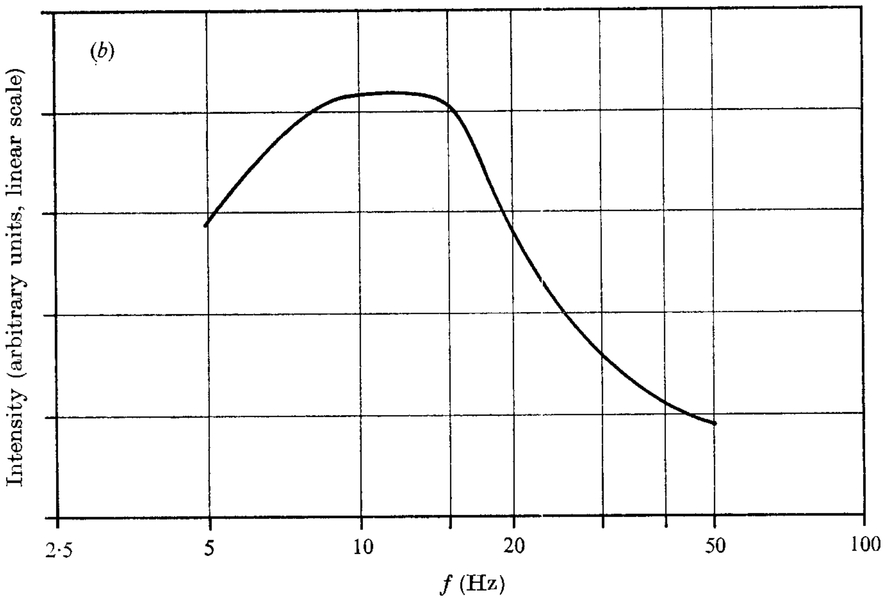


FIGURE 4. Reproduction of frequency spectra of unsteady boundary-layer separation from spheres. (*a*) Subcritical: $Re = 1{\cdot}7 \times 10^4$, hot-wire position $\phi = 75°$. (*b*) Transcritical: $Re = 3 \times 10^6$, hot-wire position $\phi = 110°$.

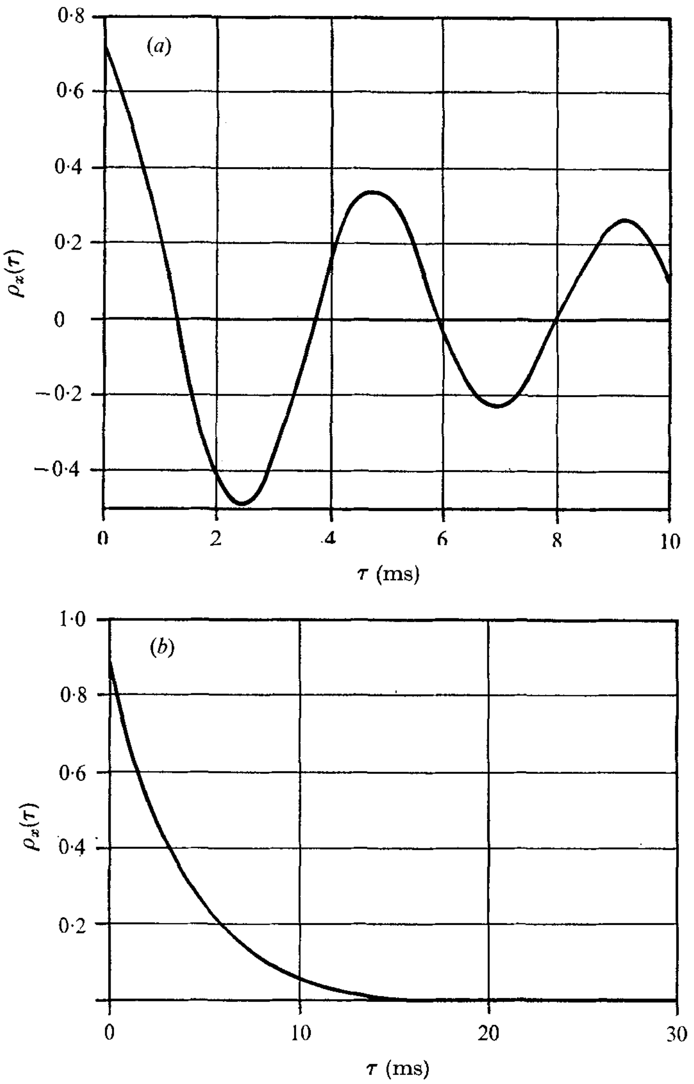


FIGURE 5. Reproduction of autocorrelations of unsteady boundary-layer separation from spheres. (*a*) Subcritical: $Re = 3 \times 10^4$. (*b*) Supercritical: $Re = 8{\cdot}5 \times 10^5$.

the shedding frequency at $Re = 3 \times 10^4$. For Reynolds numbers greater than 3×10^5 an aperiodic decrease of the autocorrelation function was found (figure 5*b*).

4. Wake configuration

4.1. *Flow visualization in a water channel*

The investigation on sphere wakes in the lower Reynolds number range was carried out in a water channel. Since the boundary layer had been coloured by dye, the wake formation could be seen optically. Complicated flow patterns,

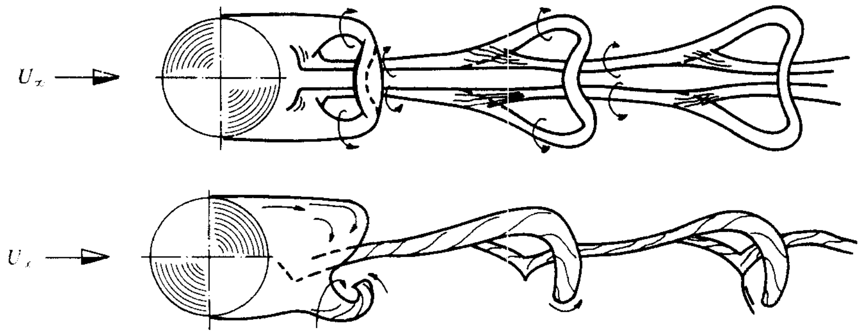


FIGURE 6. Schematic representation of the vortex configuration in the wake of spheres at $Re = 10^3$.

similar to those described, for example, by Taneda (1956), Möller (1938), Magarvey & Bishop (1961) and Magarvey & MacLatchy (1965), were observed. Permanent vortex rings, as studied by Taneda (1956), are formed below $Re = 400$. At about $Re = 400$, the separated shear layer periodically forms into vortex loops which break away. This phenomenon is described in detail and illustrated by Magarvey & MacLatchy (1965). In figure 6 the three-dimensional wake formation is represented schematically on the basis of our own observations. The flow is observed from two directions perpendicular to one another. The sketch is completed by arrows, which indicate the flow direction or the sense of the circulations.

With increasing Reynolds number the loops lose their individual character immediately after the rolling-up of the vortex sheet. They grow together and penetrate each other. The position of the rolling-up of the vortex sheet becomes closer to the sphere with increasing Reynolds number. At $Re = 400$, for instance, the length of the discontinuity sheet is about 1·5 sphere diameters, whereas at $Re = 3 \times 10^3$ the length is only 0·5 sphere diameters. Unfortunately, the maximum Reynolds number that could be reached in the water channel was only $Re = 3 \times 10^3$. Therefore, it can only be assumed that at $Re = 6 \times 10^3$, which is the lower limit of the region where unsteady-state boundary-layer separation was detected, by means of flush-mounted hot wires, the rolling-up of the shear layer occurs near the surface of the sphere. However, the experimental evidence which indicates that the Strouhal number changes by one order of magnitude when the lower critical Reynolds number is exceeded cannot at this time be explained by the author.

4.2. *Evaluation and interpretation of hot-wire signals*

In order to study the formation and structure of vortices beyond $Re = 6 \times 10^3$, two or four hot-wire probes were mounted in three different arrangements on the surface of the sphere on the circle of latitude $\phi = 75°$, ϕ being measured from the front stagnation point. At first two hot wires were installed on opposite sides on the same meridian of the sphere. As shown in figure 7(*a*), the phase angle ϵ is 180° for the two signals simultaneously recorded by the means of an oscilloscope.

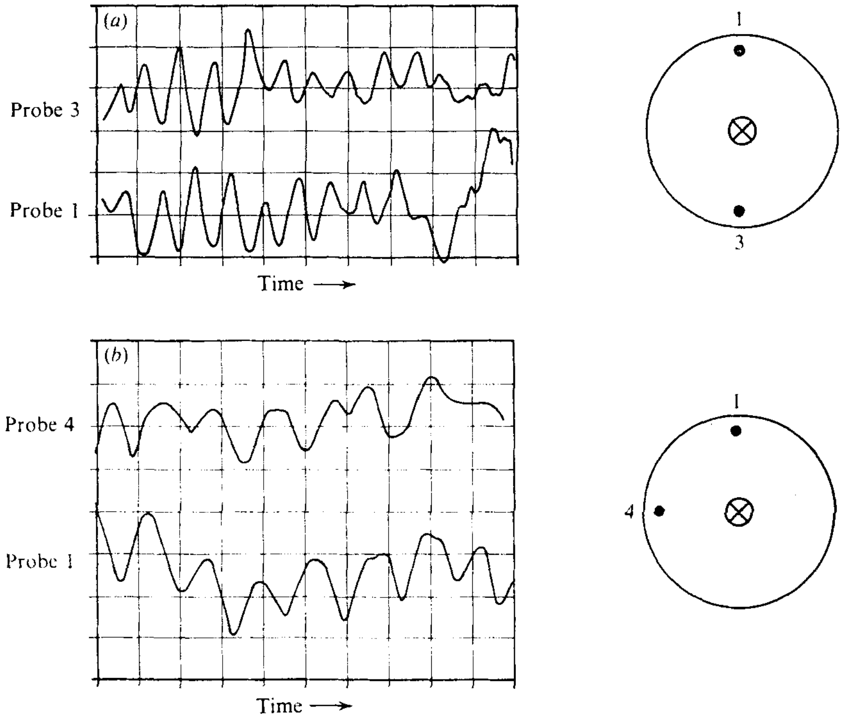


FIGURE 7. Vortex separation from spheres. Two hot-wire probes arranged as shown in sketches. (*a*) 180° arrangement. (*b*) 90° arrangement.

This phenomenon could be observed over the whole range of Reynolds numbers $6 \times 10^3 < Re < 3 \times 10^5$. In a second run, the two hot-wire probes were arranged 90° apart. Figure 7(*b*) represents the oscilloscope traces of the signals. It is obvious that the phase angle ϵ has a value of about 90°.

Finally a sphere was fitted with four hot wires, equally spaced. The four hot-wire signals were recorded simultaneously at $Re = 5 \times 10^4$ (figure 8). It is seen that the periodic event occurs in the sequence {4, 3, 2, 1}. This sequence becomes evident particularly on consideration of the disturbance starting at position 4 (trace 4, 'dist.') and propagating via 3, then 2 to position 1. The sketch in figure 8, which represents a downstream view of the sphere, illustrates the observed left-hand thread. It was noticed that the left-hand circulation was the preferred one, possibly a systematic asymmetry of the incident flow induced by the blower rotation.

In addition to using oscillographic recording, cross-correlation measurements were made. This technique enables one to obtain information on periodicity as well as on the phase angle of the two events to be compared. If t is the time, T the integration interval and τ the time delay, the cross-correlation function $R_{xy}(\tau)$ of two signals x and y is defined as

$$R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x_{(t)}\, y_{(t+\tau)}\, dt. \qquad (1)$$

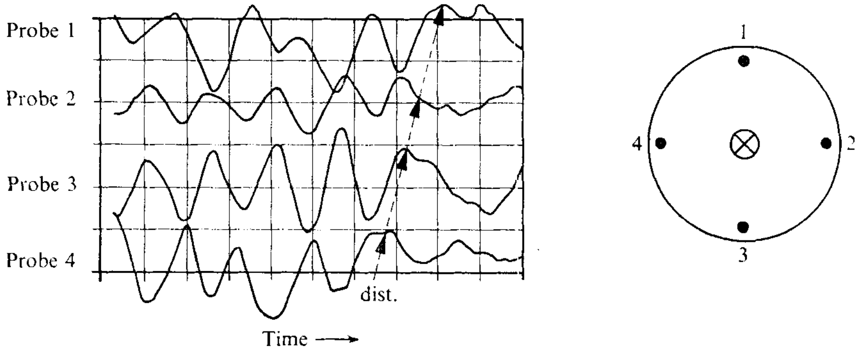


FIGURE 8. Vortex separation from spheres. Four hot wires equally spaced on the 75° circle of latitude.

If the signal x is compared with itself, the autocorrelation function $R_x(\tau)$ is obtained. Equation (1) then yields

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T x_{(t)}\, x_{(1t+\tau)}\, dt. \quad (2)$$

The product of the autocorrelation functions of the signals x and y for the time delay $\tau = 0$ is used to normalize the cross-correlation function (1) as follows:

$$\rho_{xy}(\tau) = R_{xy}(\tau)/(R_x(0)\, R_y(0))^{\frac{1}{2}}. \quad (3)$$

The value of this normalized cross-correlation function varies between $+1$ (correlated events, phase angle $\epsilon = 0$) and -1 (correlated events, phase angle $\epsilon = 180°$).

By means of the three graphs in figure 9 the compatibility of the results obtained by cross-correlation measurements with those obtained by oscillography may be demonstrated. The convention with respect to the indices of the cross-correlation function is that the signal corresponding to the second index lags behind that labelled by the first index.

The cross-correlation function ρ_{24} of the signals generated by the opposite probes 2 and 4 is given in figure 9(*a*) as a function of time delay τ. The Reynolds number is $Re = 4{\cdot}7 \times 10^4$. The correlation function starts with a negative value, which at the same time is an absolute minimum. From this it is concluded that the phase angle is $\epsilon = 180°$. The reciprocal value of the time delay, measured between two successive maxima, represents the frequency, which is common to both events. In the present case a Strouhal number of $S = 0{\cdot}181$ is calculated, which is in very good agreement with the results of the frequency analysis.

In the figures 9(*b*) and (*c*) the cross-correlograms of the signals from probes 4 and 1, and 2 and 1 are shown. Theoretically the correlation functions ρ_{41} and ρ_{21} should start at $\tau = 0$ with a value equal to zero, if the phase angle is $\epsilon = 90°$. Of course, this cannot be expected, since the shape, amplitude and frequency of the two signals are not exactly identical. However, the periodicity of the events, as well as the thread direction, can be read from the graphs. In figure 9(*b*) ρ_{41} increases with increasing time delay, whilst in figure 9(*c*) ρ_{21} decreases. Both trends are consistent with a left-hand rotation.

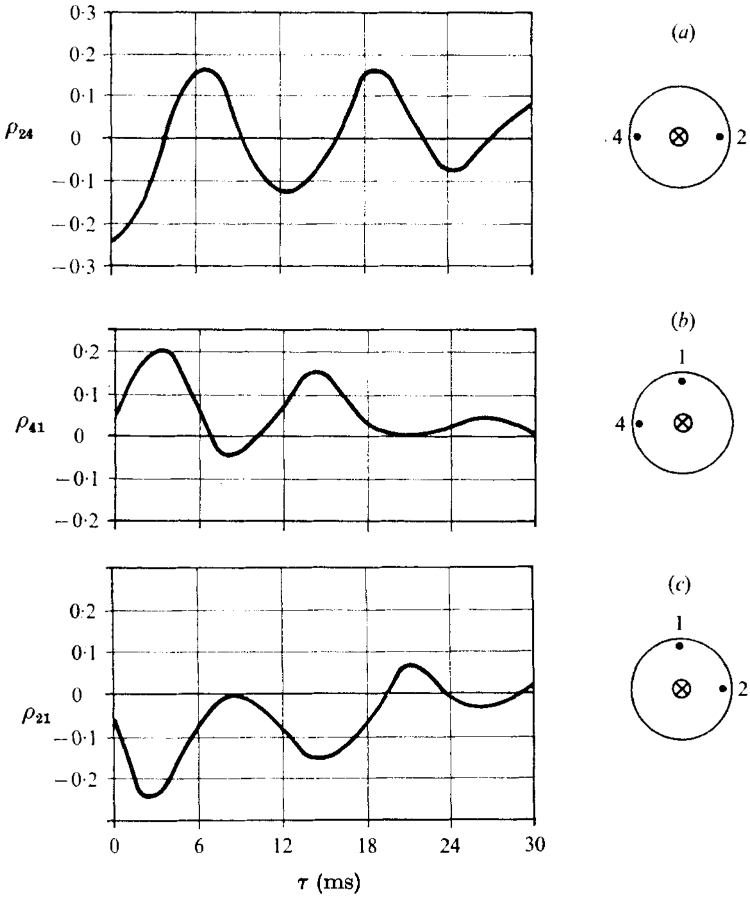


FIGURE 9. Vortex separation from spheres. Cross-correlations of two hot-wire signals for three arrangements of the probes.

The vortices cannot be released from the sphere in the form of vortex rings, as the hot-wire signals indicated a rotating separation process. It appears that the vortex separation occurs at a point and that the point of vortex release rotates around the sphere with the vortex shedding frequency. It is most inviting to assume a helical wake configuration. However, it can be demonstrated that neither the single-helix nor double-helix systems are possible wakes. Since the incident flow is free from vorticity, Thomson's circulation theorem states that the net flux of vorticity across planes perpendicular to the wake axis must also be zero. A double helix having the same circulation sense as the vortex filaments, as described for instance by Foch & Chartier (1935), as well as the double helix with circulation of opposite sense yield after resolution of the eddy vectors, through their components parallel to the wake axis, a vortex pair of the same circulation sense in the plane normal to the wake axis. Thus the net flux of vorticity is not zero.

Rosenhead (1953) suggested in a paper on vortex systems in wakes that the three-dimensional wake of the flow past bluff bodies consists of a sequence of irregularly shaped vortex loops. Their plane of orientation is purely random and is determined by the position of the point where the vortex sheet starts to roll up. This statement was made on the basis of the experimental work of Marshall & Stanton (1931), who investigated the wake of a normal disk at $Re \approx 200$.

Furthermore, Rosenhead reported that with increasing Reynolds number the vortex loops diffuse rapidly. This phenomenon was also observed in the present tests carried out in the water flow. Finally, without fixing a range of Reynolds numbers, he said that no systematic arrangement of vortices can be seen. Probably the present hot-wire tests have been conducted at Reynolds numbers still higher than those mentioned by Rosenhead. It is thus all the more surprising that, as described above, beyond the lower critical Reynolds number $Re = 6 \times 10^3$ the hot-wire signals are extremely periodic. It has been shown that the vortices cannot be formed as plane vortex rings or helical configurations. However, the information obtained from the hot-wire measurements is not yet sufficient to describe the wake structure. Therefore it would be desirable to continue the investigation of this phenomenon.

5. Final remarks

In the experiments carried out in the water channel, a test section with a rectangular cross-section $0{\cdot}14 \times 0{\cdot}15$ m was used initially. It was found that the experimental results plotted as Strouhal number *vs.* Reynolds number were a function of the sphere diameter. The results for the sphere with $d = 0{\cdot}04$ m were about 100 % higher and those for the sphere with $d = 0{\cdot}02$ m nearly 50 % higher than the values reported by Möller (1938). It was conjectured that this effect was due to tunnel blockage and a non-uniform incident velocity profile. Enlargement of the flow area and replacement of the rectangular cross-section by a circular one brought the curves together and reduced the Strouhal numbers to Möller's values. At first sight the approximately 100 % tunnel blockage seems to be unusually high. However, it must be noticed that for the Reynolds number range in question the experimental curve S *vs.* Re has a slope of about $n = 1{\cdot}5$. This means that the vortex shedding frequency depends on the velocity U_∞ to the power $m = 2{\cdot}5$. Variations of U_∞ by 30 % therefore cause errors in the relationship S *vs.* Re of about 100 %.

On the other hand no effect of blockage could be observed for the largest test sphere (0·2 m) mounted downstream of the nozzle of the wind tunnel (diameter 0·75 m). This evidence seems to be due to the fact that the jet flow is not limited by solid walls. Therefore the jet can expand, which obviously leads to a similar velocity distribution near the sphere as is found for a sphere in an infinite flow.

Cometta (1957) did not detect an increase of Strouhal number in the range $6 \times 10^3 < Re < 3 \times 10^4$ like that found in the present investigation. However, since the present results, obtained with a significant variation of the length scale, collapse in the dimensionless presentation, it is rather unlikely that significant errors occurred in determining the frequency f or the velocity U_∞ even at low blower speeds. Therefore, the increase in the Strouhal number is considered to be a physical phenomenon and not the result of deficient measurement techniques. Moreover, it is known from Roshko's (1954) experiments on vortex shedding from circular cylinders in cross-flow that the Strouhal number changes from $S = 0{\cdot}14$ at $Re = 60$ to $S = 0{\cdot}21$ at $Re = 800$. It may be an accident that the ratio of the constant Strouhal number S_c to the initial value S_i at the lowest Reynolds number is approximately the same for both the cylinder and sphere, namely $S_c/S_i \approx 1{\cdot}5$.

The question about the wake structure could not satisfactorily be answered. It seems that the vortex configurations are more complicated than appears at a first sight when considering the results of the hot-wire measurements. Further tests would be very useful.

The experiments were conducted in the laboratories of the Institut für Reaktorbauelemente, Kernforschungsanlage Jülich GmbH. The author wishes to thank Dr C. B. von der Decken, director of the Institute, for the initiative to start this research. He is also pleased to acknowledge the help of his assistants H. Gillessen, F. Hoffmanns, H. Reger, R. Rommerskirchen and W. Schmidt, who prepared and performed the tests very carefully.

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